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Enlarging the Terminal Region of Quasi-infinite Horizon NMPC Based on T-S Fuzzy Model

Shuyou Yu, Hong Chen, and Xuejun Li

Abstract: The paper presents a method for enlarging the terminal region of quasi-infinity horizon nonlinear model predictive control (NMPC) for nonlinear systems with constraints. The main technique builds on the fact that terminal controllers are fictitious and never applied to the system in the quasi-infinite horizon NMPC [1]. Based on T-S fuzzy models of nonlinear systems, we show that a parameter-dependent state feedback law exists such that the corresponding value function and its level set can be served as terminal cost and terminal region. The problem of maximizing the terminal region is formulated as a convex optimization problem based on linear matrix inequalities (LMIs). A numerical example is given to illustrate the effectiveness of the proposed method.

Keywords: Constrained nonlinear systems, linear matrix inequality (LMI), nonlinear model predictive control, terminal invariant sets, T-S fuzzy models.

1. INTRODUCTION

Model predictive control (MPC) is an effective measure to deal with multivariable constrained control problems. A control sequence is obtained by solving on-line, at each sampling instant, a finite horizon open-loop optimal control problem which uses the current state of the plant as the initial state; the first control action in this sequence is applied to the plant. Because it is difficult, if not impossible, to get an analytical solution for constrained nonlinear optimal control problem by solving HJB equations, MPC has aroused much interest in both academic community and industrial society.

One of the main results in the stability issues of MPC is the quasi-infinite horizon approach [1,2]. The quasi-infinite horizon NMPC (QIH-NMPC) needs to calculate a terminal cost, a terminal constraint region, and a terminal controller off-line, which are the essential ingredients to achieve stability [2]. The terminal controller is not applied to the plant, but just employed to calculate the terminal cost and terminal region. The terminal cost is a local control Lyapunov function and satisfies a HJB inequality in the terminal region. The terminal region, a level set of the terminal cost function, is positively invariant and renders all time-domain constraints satisfied. A remainder issue for QIH-NMPC is how to enlarge the terminal region, since the size of the terminal region affects directly the size of the domain of attraction for the nonlinear optimization problem. Moreover, the larger the terminal region, the shorter the control horizon one can choose, which reduces the on-line computational burden.

Many efforts have been made on determining the terminal penalty term and the associated terminal controller so as to enlarge the terminal region. For the case of constrained linear systems, [3] figure out terminal region by considering a saturated local control law. For nonlinear systems, using either local polytopic LDI representation [4] or local norm-bounded LDI representation [5], a terminal region is obtained by solving off-line an LMI optimization problem. In [6], a local LDI representation is used as well, and a polytopic terminal region and an associated terminal penalty are computed. The MPC formulation is modified in [7], replacing the terminal constraint with a contractive constraint provided by a sequence of reachable sets to a given invariant set.

We present a method of enlarging the domain of terminal region based on T-S fuzzy models of nonlinear systems; the resultant terminal feedback control law is parameter-dependent state feedback law. Compared with the use of time invariant linear state feedback law, the proposed approach provides the freedom in the choices of the terminal region and terminal cost needed for asymptotic stability, results in a much larger terminal region. The layout of this paper is as follows. In Section 2, the QIH-NMPC scheme is briefly introduced. Terminal region of QIH-NMPC based on T-S fuzzy models is proposed in Section 3. Simulation results are reported and discussed in Section 4.

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Shuyou Yu and Hong Chen are with the Dept. Control Science and Engineering, Jilin University, Renmin Street 5988, Changchun 130025, China (e-mails: yushuyou@126.com, chenh @jlu.edu.cn).

Xuejun Li is with the Electronic Engineering College, Changchun University, Weixing Road 6543, Changchun 130022, China (e-mail: yulgju@126.com).

2. PRELIMENARY

Consider smooth nonlinear control system:

$$\dot{x} = f(x(t), u(t)),$$
$$z(t) = g(x(t), u(t)),\ t \geq t_0,\ x(t_0) = x_0, \tag{1}$$

subject to

$$z(t) \in Z,\ \forall t \geq t_0, \tag{2}$$

where $x(t) \in R^n$, $u(t) \in R^m$ are the state and input vector, $z(t) \in Z \subset R^p$ is the output vector. Denote X and U as the projection of output space Z to the state space and input space, respectively.

Fundamental assumptions of (1) are [1,2]:

A0) nonlinear system f is twice differentiable continuous and satisfies $f(0,0) = 0$;

A1) the system [1] has a unique solution for anyinitial condition $x_0 \in X$ and any piecewise right continuous input function $u(\cdot):[0, T_P] \to U$;

A2) $U \in R^m$ is compact, $X \subseteq R^n$ is connected and (0,0) is contained in the interior of $X \times U$.

For actual state $x(t)$, the optimization problem in QIH-NMPC is formulated as follows [1,9]:

$$\min_{\bar{u}(\cdot)} J(x(t), \bar{u}(\cdot)) \tag{3}$$

subject to

$$\dot{\bar{x}} = f(\bar{x}, \bar{u}),\ x(t; x(t), t) = x(t)$$
$$\bar{z}(\tau) \in Z,\ \tau \in [t, t+T_p],$$
$$x(t+T_p; \bar{x}(t), t) \in \Omega,$$

with $J(x(t), \bar{u}(\cdot)) = \int_t^{t+T_P} l(\bar{x}(t+\tau; x(t)), \bar{u}(t+\tau; x(t)))d\tau$ $+V(x(t+T_P))$. T_P is prediction horizon, $\bar{x}(\cdot; x(t), t)$ denotes the state trajectory starting from $x(t)$ under control $\bar{u}(t)$, $l(\cdot,\cdot)$ is stage cost such that:

A3) $l(x,u): X \times U \to R$ is continuous, satisfies $l(0,0) = 0$ and $l(x,u) > 0\ \ \forall (x,u) \in X \times U \setminus \{0,0\}$.

$\Omega(\alpha)$ is a neighborhood of origin and defines a level set of a positive definite function $V(\cdot)$

$$\Omega(\alpha) := \{x \in R^n \mid V(x) \leq \alpha, \alpha > 0\}. \tag{5}$$

Moreover, $\Omega(\alpha)$ and $V(x)$ are said to be terminal region and terminal penalty if there exists a continuous local controller $u = \kappa(x)$ such that

B0) $\Omega(\alpha) \subseteq X$,

B1) $g(x, \kappa(x)) \in Z$, for all $x \in \Omega(\alpha)$,

B2) $V(x)$ satisfies the following HJB inequalities

$$\frac{\partial V(x)}{\partial x} f(x, \kappa(x)) + l(x, \kappa(x)) \leq 0, \tag{6}$$

$\Omega(\alpha)$ has the following additional properties [9]:

- the point $0 \in R^n$ is contained in the interior of $\Omega(\alpha)$ due to the positive definiteness of $V(x)$ and $\alpha > 0$,
- $\Omega(\alpha)$ is closed and connected due to the continuity of V in x,
- $\Omega(\alpha)$ is positive invariant for the nonlinear system (1) controlled by $u = \kappa(x)$.

Then, following stability results can be gained:

Lemma 1 [9]**:** Suppose that

(a) assumptions A0)-A3) are satisfied,

(b) for the nonlinear system (1), there exists a locally asymptotically stabilizing controller $u = \kappa(x)$, a continuously differentiable, positive definite function $V(x)$ that satisfies (6), and a terminal region $\Omega(\alpha)$ defined by (5),

(c) the optimal control problem described by (3) is feasible at time $t = 0$.

Then, for sufficient small sampling time δ, the closed-loop system is nominally asymptotically stable with the region of attraction D being the set of all states for which the open-loop optimal control problem has a feasible solution.

3. TERMINAL REGION OF QIH-NMPC BASED ON T-S FUZZY MODEL

It has been proven any twice differentiable continuous nonlinear function can be approximated to any degree of accuracy using linear T-S fuzzy model [8]. In this section, we will introduce the construction procedures of T-S fuzzy model for specified nonlinear system. Then a model-based fuzzy terminal controller design and terminal penalty using the concept of "parallel distributed compensation" is described. The related synthesis problems are formulated as LMI problems.

3.1. T-S fuzzy model

The constraints under consideration are given by:

$$-\hat{z}_k \leq z_k(t) \leq \hat{z}_k,\ k = 1, 2, \ldots, p,\ \ t \geq t_0, \tag{7}$$

where $z_k(\cdot)$ is k th element of outputs, $\hat{z}_k$ is a positive scale. In the paper, we choose $l(x,u) = x^T Q x + u^T R u$, $Q \geq 0$, $R > 0$, and $\Omega(\alpha, P) := \{x \in R^n \mid x^T P x \leq \alpha,\ \alpha > 0\}$.

The ith rule of the T-S fuzzy model is

$$\begin{aligned} R^i:\ & \text{IF } \lambda_1(t) \text{ is } M_1^i, \ldots, \text{and } \lambda_r(t) \text{ is } M_2^r, \\ & \text{Then } \dot{x}(t) = A_i x(t) + B_i u(t),\ x(t_0) = x_0, \\ & z(t) = C_i x(t) + D_i u(t),\ t \geq t_0,\ i = 1, 2, \ldots, r. \end{aligned} \tag{8}$$

Here R^i represents ith fuzzy rule, M_j^i is the fuzzy set and r is the number of model rules. $\lambda_1(t), \ldots, \lambda_r(t)$

are known premise variables that may be functions of the state variables, external inputs, and/or time. $\lambda(t) \in R^r$ denotes the vector containing all the individual elements of $\lambda_1(t), \lambda_2(t), \ldots, \lambda_r(t)$. Then, nonlinear system (1) can be approximated as T-S fuzzy models,

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\lambda)[A_i x(t) + B_i u(t)], \tag{9a}$$

$$z(t) = \sum_{i=1}^{r} h_i(\lambda)[C_i x(t) + D_i u(t)], \tag{9b}$$

where $h_i(\lambda(t)) = \mu_i(\lambda(t)) / \sum_{i=1}^{r} \mu_i(\lambda(t))$, and $\mu_i(\lambda(t)) = \prod_{i=1}^{r} M_j^r(\lambda_j(t))$, the term $M_j^r(\lambda_j(t))$ is the grade of membership of $\lambda_j(t)$ in M_j^r. Since $\mu_i(\lambda(t)) \geq 0$, we have $h_i(\lambda(t)) \geq 0$, $\sum_{i=1}^{r} h_i(\lambda(t)) = 1$.

We utilize the concept of parallel distributed compensation [8] to design fuzzy controllers for fuzzy system(1). The idea is to design a compensator for each rule of the fuzzy model, and the designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts

$$\begin{aligned} &\text{C}^{\text{i}}: \text{IF} \quad \lambda_1(t) \text{ is } M_1^i, \ldots, \text{and } \lambda_{\text{r}}(t) \text{ is } M_2^r, \\ &\quad \text{Then } \text{u}(t) = K_i x(t), \; i = 1, 2, \ldots, r, \end{aligned} \tag{10}$$

where $K_i \in R^{m_u \times n}$ is a constant feedback matrix. The resulting overall fuzzy controller, which is parameter dependent in general, is a fuzzy blending of each individual controller

$$u(t) = \kappa(\lambda(t)) x(t), \tag{11}$$

where $\kappa(\lambda(t)) = \sum_{i=1}^{r} h_i(\lambda(t)) K_i$. Substituting (11) into (9), we obtain closed-loop system model

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i(t)[A_i + B_i \sum_{j=1}^{r} h_j(\lambda(t)) K_j] x(t), \\ &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(t) h_j(t)(A_i + B_i K_j) x(t), \end{aligned} \tag{12a}$$

$$z(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(t) h_j(t)(C_i + D_i K_j) x(t). \tag{12b}$$

Usually, the synthesis and analysis of the fuzzy control system can be reduced to a feasibility problem of parameter matrices inequalities [8]. Here we introduce a less conservative feasibility result of the parameter matrices inequalities that can be solved effectively by LMIs.

Lemma 2 [10]**:** If there exists matrices $W_{ii} = W_{ii}^T$, $W_{ij} = W_{ji}^T \; (i \neq j)$ such that $Q_{ij} \; (1 \leq i, j \leq r)$, satisfies $Q_{ii} \leq W_{ii} \; (i = 1, 2, \ldots, r)$, $[W_{ij}]_{r \times r} \leq 0$, $Q_{ij} + Q_{ji} \leq W_{ij} + W_{ji} (j < i)$, then parameter matrices inequalities $\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(t) h_j(t) Q_{ij} \leq 0$ are feasible, where $h_i(\lambda(t)) \geq 0$, $\sum_{i=1}^{r} h_i(\lambda(t)) = 1$, $\forall \lambda(t)$, $[W_{ij}]_{r \times r} = \begin{pmatrix} W_{11} & \cdots & W_{1r} \\ \vdots & \vdots & \vdots \\ W_{1r} & \cdots & W_{rr} \end{pmatrix}$.

Remarek 1: For the cases of "$<$", "$>$" and "$\geq$", we have the similar results.

3.2. Satisfaction of HJB inequalities

Based on the T-S fuzzy model of nonlinear system (1), the HJB inequalities conditions (6) can be formulated ultimately as a LMI problem. This is attractive since efficient computational methods for solving such problems are available [11] .

Theorem 1: For system (12), if there exists positive definite matrix X and matrices Y_j such that

$$\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\lambda(t)) h_j(\lambda(t)) \begin{bmatrix} \Gamma + \Gamma^T & X & Y_i^T \\ X & -Q^{-1} & 0 \\ Y_j & 0 & -R^{-1} \end{bmatrix} \leq 0$$

with $\Gamma = A_i X + B_i Y_j$. Then, $V(x) = x^T P x$ with $P = X^{-1}$ renders the HJB inequalities (6) satisfied, and $\kappa(\lambda(t)) = \sum_{j=1}^{r} h_j(\lambda(t)) K_j$ with $K_j = Y_j X^{-1}$ is the parameter-independence controller.

Proof: By the Schur complement, it follows that the matrix inequalities (15) is equivalent to

$$\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\lambda) h_j(\lambda)(\Gamma + \Gamma^T + XQX + Y_i^T R Y_j) \leq 0,$$

It is equivalent to the existence of $P = X^{-1}$, and $K_j = Y_j X^{-1}$ such that

$$\begin{aligned} &\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\lambda) h_j(\lambda)((A_i + B_i K_j)^T P + P(A_i + B_i K_j) \\ &\qquad + Q + K_i^T R K_j) \leq 0. \end{aligned} \tag{16}$$

We choose $V(\xi) = \xi^T P \xi$ as a Lyapunov candidate, the time derivative of $V(x)$ along the trajectory of (12) is given as follows:

$$\begin{aligned} \frac{dV(x)}{dt} &= \dot{x}(t) P x(t) + x(t) P \dot{x}(t) \\ &= x^T(t) A_{cl}^T(\lambda(t)) P x(t) + x^T(t) P A_{cl}(\lambda(t)) x. \end{aligned} \tag{17}$$

By (16), we have

$$\frac{dV(x)}{dt} \le -x^T(t)\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\lambda)h_j(\lambda)(Q+K_i^T QK_j)\}x(t)$$
$$= -x^T(t)Qx(t) - \kappa^T(x)R\kappa(x). \tag{18}$$

then HJB inequalities (6) hold, $\kappa(\lambda(t))$ is an associated controller.

3.3. Constraints satisfaction

We will discuss the conditions that system satisfies output constraints (7) under the controller $\kappa(\lambda(t))$ while the system states enter into the region $\Omega(\alpha)$.

Theorem 2: If X and Y_j $(j=1,2,\ldots,r)$ satisfy (15) and furthermore the matrix inequalities

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\lambda(t))h_j(\lambda(t))\begin{bmatrix} \frac{1}{\alpha}\hat{z}_k^2 & e_k^T(C_iX+D_iY_j) \\ * & X \end{bmatrix} \ge 0,$$
$$k=1,2,\ldots,p, \tag{19}$$

where e_k is k th element of basis vector in the constraint vector space, then for $\forall x(t)\in\Omega(P,\alpha)$, controller $\kappa(\lambda(t))=\sum_{j=1}^{r} h_j(\lambda(t))K_j$ drives system (12) satisfying the constraint (7).

Proof: By the use of (12b), the satisfaction of constraint (7) requires $x(t)^T(C_{cl}^T(\lambda(t))e_k e_k^T C_{cl}(\lambda(t)))\cdot x(t)\le \hat{z}_k^2$. Due to $x(t)\in\Omega(P,\alpha)$, which holds if

$$\frac{x(t)^T(C_{cl}^T(\lambda(t))e_k e_k^T C_{cl}(\lambda(t)))x(t)}{\hat{z}_k^2} \le \frac{x^T(t)Px(t)}{\alpha}. \tag{21}$$

For any $x(t)$, (21) holds by enforcing

$$\frac{P}{\alpha} - \frac{(C_{cl}^T(\lambda(t))e_k e_k^T C_{cl}(\lambda(t)))}{\hat{z}_k^2} \ge 0. \tag{22}$$

By Schur complement, the matrix inequality (22) is equivalent to

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\lambda(t))h_j(\lambda(t))\begin{bmatrix} \frac{1}{\alpha}\hat{z}_k^2 & e_k^T(C_i+D_iK_j) \\ * & P \end{bmatrix} \ge 0. \tag{23}$$

If we perform a congruence transformation with diag{X,I} on both sides of (23), we obtain the required (19).

3.4. Calculating terminal region

If there exists positive definite matrix X, matrices Y_j, and a scale $\alpha>0$, independence of unknown parameter vector $h_i(\lambda)$, satisfying (15) and (19), then $\Omega(\alpha,P)$ is the terminal fuzzy model. Furthermore, because T-S fuzzy system can approximate nonlinear system to any degree of accuracy, $\Omega(\alpha,P)$ is also the terminal region of nonlinear system (1).

We define $X_0=\alpha X$, $Y_{j0}=\alpha Y_j$, inequality constraints (15), (19) can be rewritten as

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\lambda(t))h_j(\lambda(t))L_{ij}\le 0, \tag{24}$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\lambda(t))h_j(\lambda(t))F_{ij}\ge 0, \tag{25}$$

where

$$L_{ij}= \begin{bmatrix} A_iX_0+B_iY_{j0}+(A_iX_0+B_iY_{j0})^T & X_0 & Y_{i0}^T \\ X_0 & -\alpha Q^{-1} & 0 \\ Y_{j0} & 0 & -\alpha R^{-1} \end{bmatrix},$$

$$F_{ij}=\begin{bmatrix} \hat{z}_k^2 & e_k^T(C_iX_0+D_iY_{j0}) \\ * & X_0 \end{bmatrix}.$$

Constraints (24), (25) are parameter dependent, we can recast them as LMI by Lemma 2. That is, (24) and (25) are feasible if there exist T_{ij} and M_{ij} such that

$$\begin{cases} L_{ii}\le T_{ii}\ (i=1,2,\ldots,r), \\ L_{ij}+L_{ji}\le T_{ij}+T_{ji}(j<i), \quad [T_{ij}]_{r\times r}\le 0, \end{cases} \tag{26a}$$

$$\begin{cases} F_{ii}\ge M_{ii}\ (i=1,2,\ldots,r), \\ F_{ij}+F_{ji}\ge M_{ij}+M_{ji}(j<i), \quad [M_{ij}]_{r\times r}\ge 0. \end{cases} \tag{26b}$$

Let $\Omega(\alpha,P)$, also referred to as $\Omega(1,X_0^{-1})$, denotes the ellipsoid centered at the origin determined by P. The volume of Ω is proportional to $\det(X_0)$, $X_0=\alpha P^{-1}$ [11]. The objective function $\det(X_0)$ is not convex, but monotonic transformations can render this problem to LMI. Here, we adopt the scheme proposed by [12], and maximization volume of the ellipsoid Ω can be reformulated as the following LMI optimization problem

$$\max_{\alpha,X_0,Y_{j0}} (\det X_0)^{\frac{1}{n}}$$
$$\text{s.t.} \quad \alpha>0,\ X_0>0,\ \text{(26a) and (26b)}. \tag{27}$$

Sometimes solving the optimization problem (27) gives a very large terminal matrix (in the sense of norm) such that the effect of the integration term in (3) almost disappears. A very strong penalty on the terminal states may have a bad influence on the achievement of the control performance which is specified by the finite horizon cost [1]. The trade off between a large terminal region and good achievement of the desired control performance can be made by limiting the norm of the matrix P [4]. Because of $P = X^{-1} = \alpha X_0^{-1}$, this can be achieved by the requirement that α be less than or equal to a given constant.

4. A NUMERICAL EXAMPLE

Consider the system used in [1] described by

$$\begin{aligned}\dot{x}_1 &= x_2 + u(\mu + (1-\mu)x_1),\\ \dot{x}_2 &= x_1 + u(\mu - 4(1-\mu)x_2).\end{aligned} \tag{28}$$

As pointed out in [1], this plant is unstable and its linearized system is stablizable (but not controllable) for $\mu \in (0,1)$. Assume that x_1 and x_2 are observable, and the control constrained are $-2 \le u(t) \le 2$. The stage cost is given by $l(x,u) = x^T Qx + u^T Ru$, $Q = \text{diag}\{0.5\ \ 0.5\}$, $R = 1$. We can write (28) in the following space representation

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}(1-\mu)u & 1\\ 1 & -4(1-\mu)u\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\mu\\ \mu\end{bmatrix}u. \tag{29}$$

Therefore, by the same way given by [8], we obtain the following fuzzy model which exactly represents the nonlinear equation under $u(t) \in [-2,2]$:

Rule 1: $\begin{cases}\text{If } u(t) \text{ is } M_1,\\ \text{Then } \dot{x}_1 = A_1 x + B_1 u\end{cases}$ Rule 2: $\begin{cases}\text{If } u(t) \text{ is } M_2,\\ \text{Then } \dot{x}_2 = A_2 x + B_2 u\end{cases}$

Here,

$$x(t) = [x_1(t)\quad x_2(t)]^T,\ \lambda(t) = u(t),$$

$$M_1(u(t)) = \frac{1}{2}(1 + \frac{u(t)}{d}),\ M_2(u(t)) = \frac{1}{2}(1 - \frac{u(t)}{d}),$$

$$A_1 = \begin{bmatrix}(1-\mu)d & 1\\ 1 & -4(1-\mu)d\end{bmatrix},$$

$$A_2 = \begin{bmatrix}-(1-\mu)d & 1\\ 1 & 4(1-\mu)d\end{bmatrix},$$

$$B_1 = B_2 = [\mu\quad \mu]^T.$$

While μ is chosen as 0.8, solving the LMI optimization problem (27), we get

$$P = 10^5 \times \begin{bmatrix}0.5109 & 0.5908\\ 0.5908 & 1.3634\end{bmatrix},\ \alpha = 1.7120 \times 10^5.$$

The associated terminal region yielded is shown in Fig. 1 by a solid ellipsoid, and terminal region given by [1] is shown by a dashed ellipsoid.

Compared with the state and control penalty matrices Q and R in stage cost $l(x,u)$, the norm of the terminal penalty matrix P yielded by the optimization problem (27) is too large. To avoid this, we can introduce an additional constraint $\alpha \le 10$. Solving optimization problem (27) with it we get $P = \begin{bmatrix}4.0608 & 4.4021\\ 4.4021 & 9.9842\end{bmatrix}$ and the associated terminal region is shown in Fig. 2 as a solid ellipsoid. It is slightly smaller than the terminal region given by optimization without the constraint

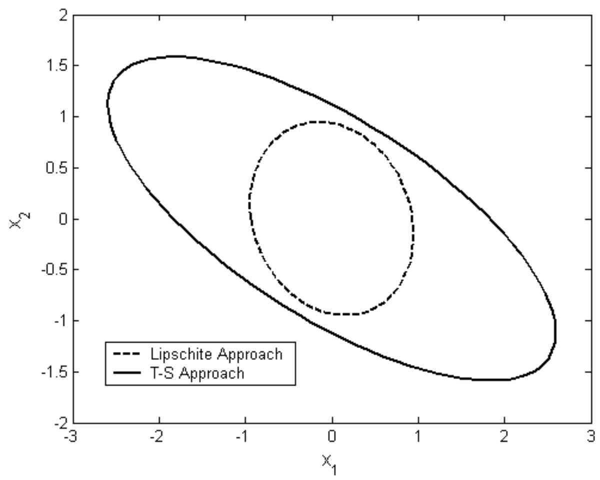

Fig. 1. Comparison of the terminal region.

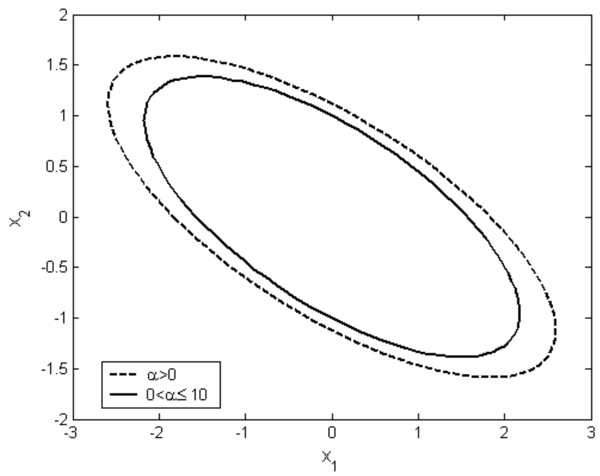

Fig. 2. With and without the constraint $\alpha \le 10$.

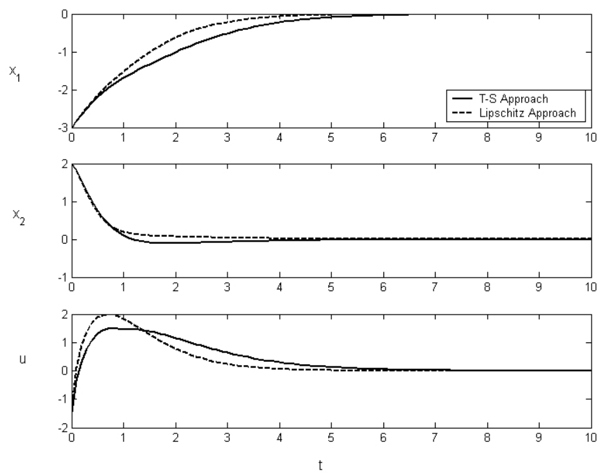

Fig 3. Comparing of dynamic response.

$\alpha \le 10$ (shown in dashed ellipsoid), but the norm of penalty matrix is greatly reduced.

To the initial state $(-3, 2)$, if we use the terminal region and terminal penalty given by [1], the optimization problem will not feasible until the control horizon N increases to 14. However, using the terminal region and terminal penalty gaining through solving the optimization problem (27), imposing the constraint $\alpha \le 10$, it is feasible while $N = 5$. Fig. 3 show the time profiles for the closed loop for the two cases (the control and predictive horizon are $N = 14$ for Lipschite approach, and $N = 5$ for T-S approach; the sampling time is $\delta = 0.1$. Obviously, the latter requires a significant smaller amount of computations and its performance reduction is slightly. It can be seen also that the constraint is not violated.

5. CONCLUSIONS

In the paper, we present a method for enlarging the terminal region of model predictive control for nonlinear systems with constraints. We highlight the fact that "fictitious" terminal controller is never applied to the system, but used to choose the terminal penalty term and determine the terminal region in QIN-NMPC. Based on T-S fuzzy models of nonlinear systems, we show that a parameter-dependent state feedback law exists such that the corresponding value function and its level set can be served as terminal cost and terminal region. It is shown further that the above issue can be reformulated as a well-defined convex optimization problem, and can be solved by linear matrix inequalities. Compared with the use of time invariant linear state feedback control law, parameter-dependent state feedback law results in a much larger terminal region, which is confirmed by a numerical example.

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Shuyou Yu received the B.S. degree from Jilin University of Technology in 1997, and the M.S. degree from Jilin University in 2005. Now he is pursing a Ph.D. degree at Jilin University. His research interests include the theory and application of model predictive control, robust control.

Hong Chen received the B.S and M.S. degrees in Process Control from the Zhejiang University, China, in 1983 and 1986 respectively, and the Ph.D. degree from the University of Stuttgart, Germany, in 1997. In 1986, she joined the Jilin University of Technology, China. From 1993 to 1997, she was a "wissenschaftlicher Mitarbeiter" at the Institut fuer Systemdynamik und Regelungstechnik, University of Stuttgart. Since 1999, she has been a Professor at the Jilin University. Her current research interests include model predictive control, optimal and robust control, applications in process engineering and mechatronic systems.

Xuejun Li received the Ph.D. degree in Control Science and Engineering from Jilin University in 2007. She is currently an Assistant Professor in Institute of Electric Information Engineering, at Changchun University. Her research interest covers robust control and delay system control.